THE SHAPE OF A WATER VORTEX

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Introduction

The shape of a water vortex has been investigated in relation to path curves by Lawrence Edwards and John Blackwood (Edwards 1993) where it was found that a path curve with $\lambda = -1.75$ fitted the whole form well provided the path curve had one invariant point at infinity. Since then Georg Sonder (Sonder 19??) has studied them more closely and found that a better fit occurs near the top of the vortex with $\lambda = -2.9$. In this article a method based on Legendre Polynomials will be described where the shape harmonics to fit a path curve vortex were investigated, and a surprising relation to egg path curves and also pivot-transforms was discovered. This arose from investigations in connection with counterspace to relate the cosmos to the earthly, by identifying the shape harmonics of the vortex with surface spherical harmonics, the latter in connection with the tone ether (see Thomas 1999 for an introduction to the latter). It was found that shapes relating to practical vortices, e.g. in stirring biodynamic preparations, could then also be obtained.

Legendre Polynomials

Clearly an account of the complete theory underlying Legendre polynomials is beyond the scope of a brief article (see e.g. Sneddon 1956 for that), but the following 'skeleton key' may be of service. Just as Fourier transforms expand time waveforms into harmonics, so Legendre polynomials expand forms or shapes into what we may call shape harmonics. Such methods have been used extensively e.g. in the investigation of the shape of the Earth using observations of artificial satellites (King-Hele). Such methods require a complete set of orthogonal functions to be devised to describe the harmonics, 'orthogonal' meaning in this context that variation of one harmonic does not affect the contributions of the others. Setting aside the proofs (*op cit*), Legendre showed that the following polynomials are orthogonal and may be used for analysing shapes:

$$P_0(\mu) = 1$$
$$P_1(\mu) = \mu$$
$$P_2(\mu) = \tfrac{1}{2}(3\mu^2-1)$$
$$P_3(\mu) = \tfrac{1}{2}(5\mu^3-3\mu)$$
$$(n+1)P_{n+1}(\mu) = (2n+1)\mu P_n(\mu) + (n-1)P_{n-1}(\mu) \tag{1}$$

Plotting them as radial functions of θ where $\mu = \cos(\theta)$ we obtain:

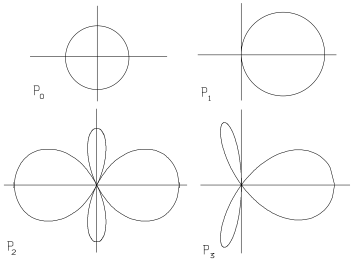

Figure 1

A shape described by a function f(μ) defined on the interval $-1 \leq \mu \leq 1$ is expanded as follows:

$$f(\mu) = \sum_{r=0}^{\infty} c_r P_r(\mu) \qquad (2)$$

where c_r are the coefficients determining the contribution of each $P_r(\mu)$. These are calculated using the fact that

$$\int_{-1}^{1} f(\mu) P_n(\mu) d\mu = \sum_{r=0}^{\infty} c_r P_r(\mu) P_n(\mu) d\mu$$

(3)

and

$$\int_{-1}^{1} P_m(\mu) P_n(\mu) d\mu = \frac{2}{2n+1} \delta_{m,n} \qquad (4)$$

which is the orthogonality property, so that the sum on the right hand side of (3) is

$$\sum c_r \frac{2}{2n+1} \delta_{r,n} = \frac{2c_n}{2n+1}$$

giving

$$c_n = \left(n + \tfrac{1}{2}\right) \int_{-1}^{1} f(\mu) P_n(\mu) d\mu$$

(5)

These are found for increasing n until a sufficiently close match is obtained in (2).

Application to Vortices

Strictly speaking the function f(μ) must be defined on the interval $-1 \le \mu \le 1$ or, for a radial function, $-\pi \le \theta \le 0$. Also it must satisfy the Dirichlet conditions on that interval which fortunately means that it need not be continuous, converging on the midpoint of the jump at a discontinuity. This enables us to expand a vortex profile, for which we must define how we describe it to obtain an interval $-1 \le \mu \le 1$. This is shown in Figure 2.=

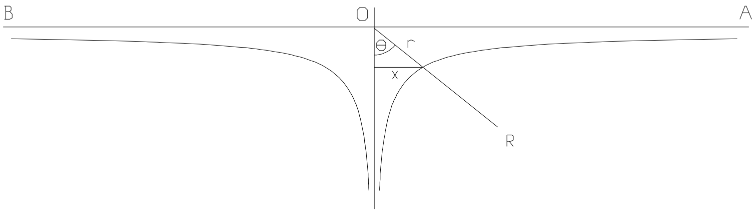

Figure 2

We rotate OR about O from OA to OB and take f(cos(θ–π)) = r in the above diagram, and expand that. r is given by x.cosec(θ) where

$$x = \exp\left\{\left(\frac{1}{1+\lambda}\right)\log(1+h)+\left(\frac{\lambda}{1+\lambda}\right)\log(1-h)\right\} \qquad (6)$$

where λ is the parameter of the vortex profile and h is the fractional height of the point on the curve within the invariant triangle (see Thomas www for the derivation of this formula, first given by Barry Christian).

Expanding the result using numerical methods to evaluate (5) and a minimum rms deviation to limit the number of terms, a vortex with λ = -2.9 was expanded into 123 terms, the result being as shown in Figure 3. The continuous curve is the ideal curve, the fitted curve being indicated by squares. The discontinuity is treated as expected theoretically, its height being that at which the two lines OR adjacent to the vertical axis intercept the vortex. The function was evaluated at 100 points in the interval which gave an angular increment of 1.8 degrees, determining the height at the discontinuity as above, and also the first and last points off the horizontal axis. As can be seen there is a slight ‘ringing’ but the fit is good. The program used indicated a best fit after 74 terms, which gave more pronounced ‘ringing’ about the curve but smaller discontinuities at the ends. 123 terms produced a much better fit overall at the expense of larger discontinuities at the ends, the rms deviation being unduly skewed by those end points.

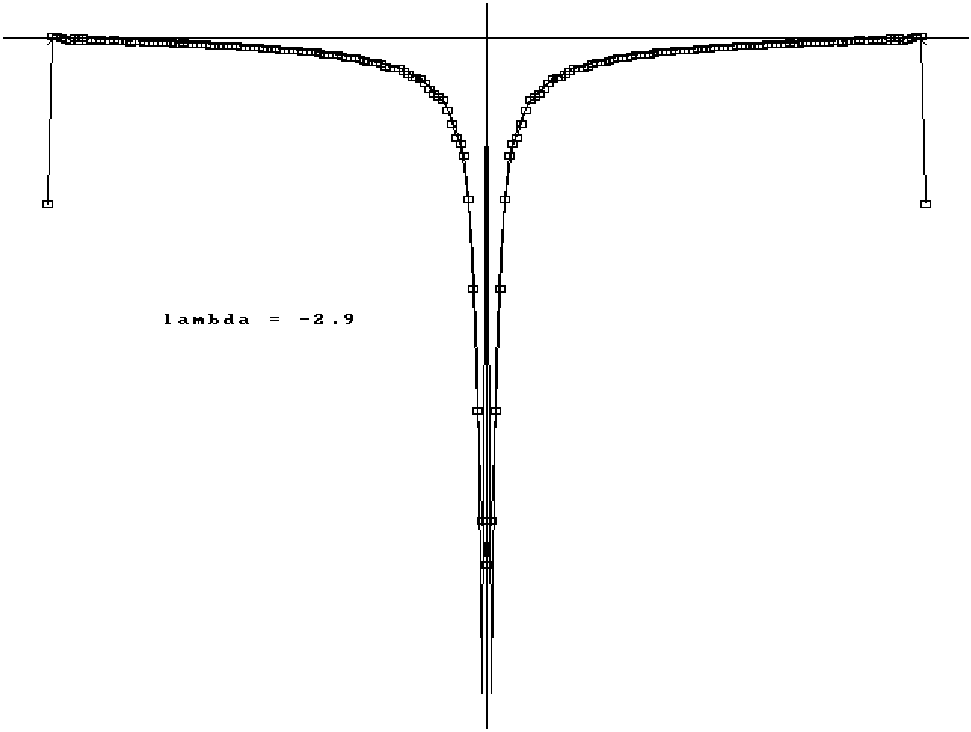

Figure 3

The harmonic envelope for the above, i.e. a graph of the coefficients c_r of the Legendre polynomials, is shown on the left in Figure 4.

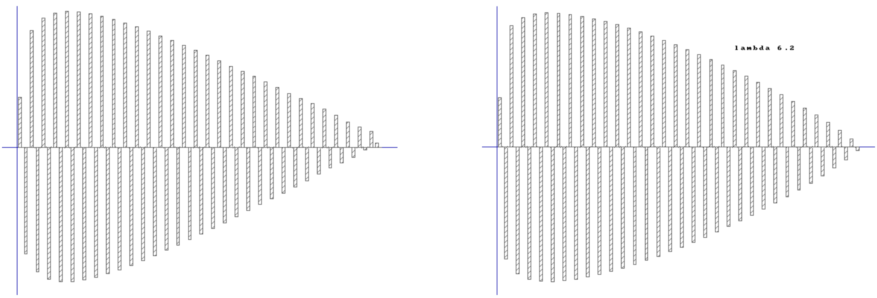

Figure 4

The coefficient number increases to the right; alternate coefficients are zero. This is very like a path curve egg form with λ=6.2. A set of harmonics was calculated with an envelope having exactly that λ, as shown on the right of Figure 4, and then the resulting vortex form was produced from it is shown in Figure 5.

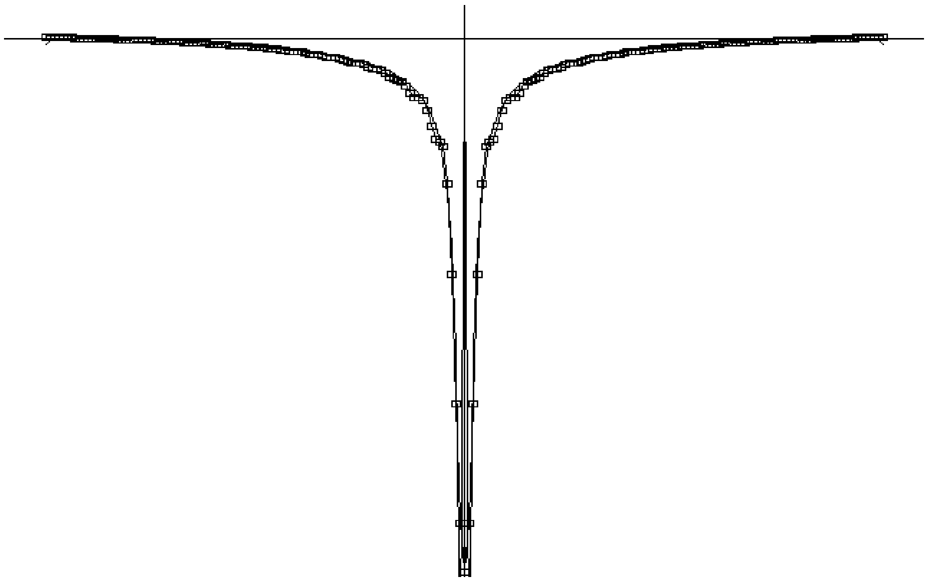

Figure 5

This is an improvement on the original with no discontinuities at the ends. *Thus a deep connection between a path curve vortex and a path curve egg is revealed, the egg determining the harmonic envelope of the vortex.* The work was repeated for a number of different λ values for the vortex with similar results, and the relation between that and the λ of the harmonic envelope was plotted as shown in Figure 6.

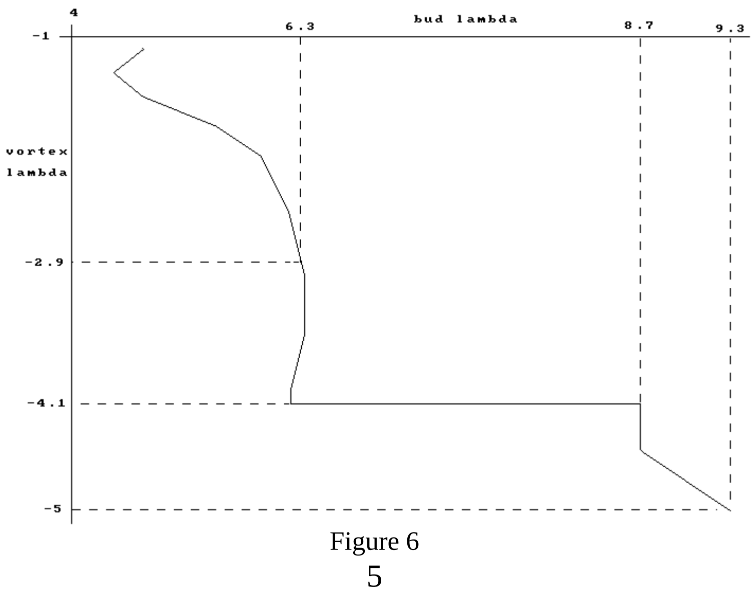

Figure 6

The λ for a true water vortex seems well placed in a ‘stable’ region with little variation in λ for the harmonic envelope (marked as ‘bud lambda’ in the diagram). The sudden discontinuity at λ = -4.1 is striking. The harmonic envelope for 123 coefficients at λ = -5 was found to be as shown in Figure 7:

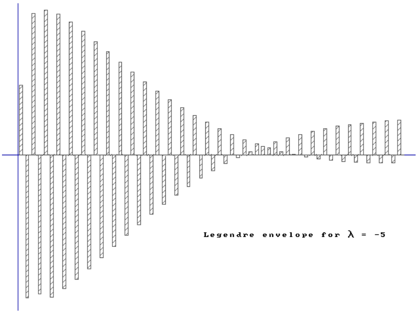

Figure 7

It seems that the profile is departing from that of an egg, and is very reminiscent of a pivot-transform profile when a water vortex is transformed (see Edwards 1993 for an account of the pivot-transform). Perhaps the discontinuity indicates a switch from egg to pivot-transform. It is tricky but possible to fit a pivot-transform profile to a given curve. Such a fit to the above envelope is shown in Figure 8 while Figure 9 shows the resulting vortex derived from it, compared with the original vortex.

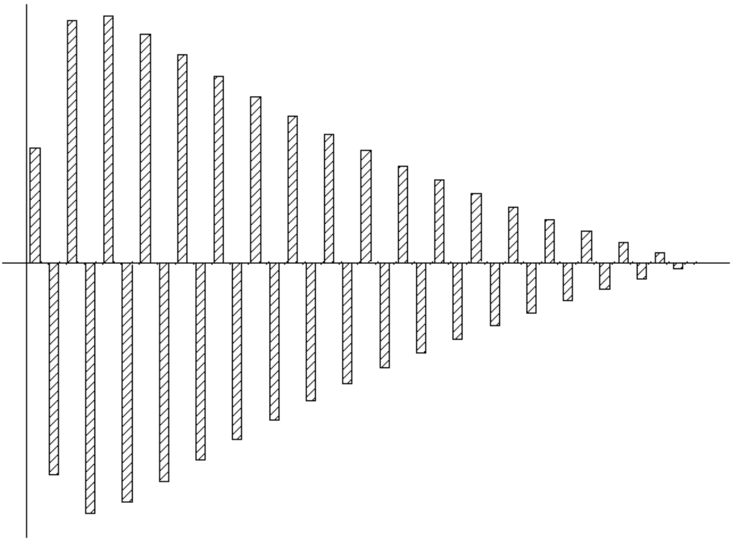

Figure 8

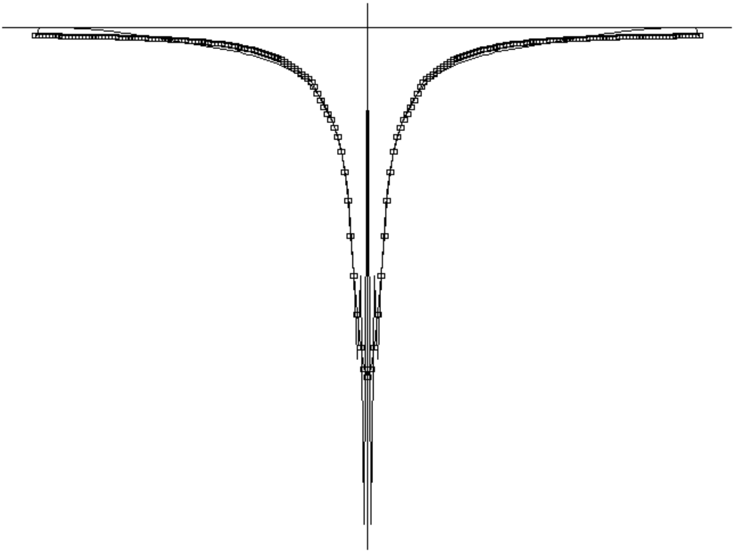

Figure 9

Although the pivot-transform extends beyond the first zero as in Figure 7, it was found that the best results were obtained taking only those Legendre coefficients before the zero as in Figure 8. The deviation at the extremities appears more like a physically obtained vortex then does the ideal.

Real Vortices

When water is stirred in a bucket, as in making a biodynamic preparation, the kind of vortex shown in Figure 2 is not at first obtained, but rather something more like that shown in Figure 10.

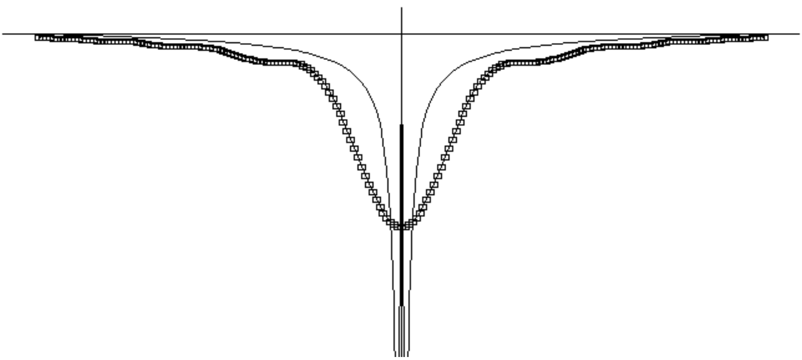

Figure 10

This arises when the egg profile of Figure 4 (right) is recalculated for 21 coefficients instead of 123 and then summed to produce a vortex. As we progressively reduce the number of coefficients below 123 in this way, but retain a complete egg profile, the vortex becomes more and more shallow. The corresponding form derived from a pivot-transform profile as in Figure 8, but with 20 instead of 73 coefficients, is shown in Figure 11.

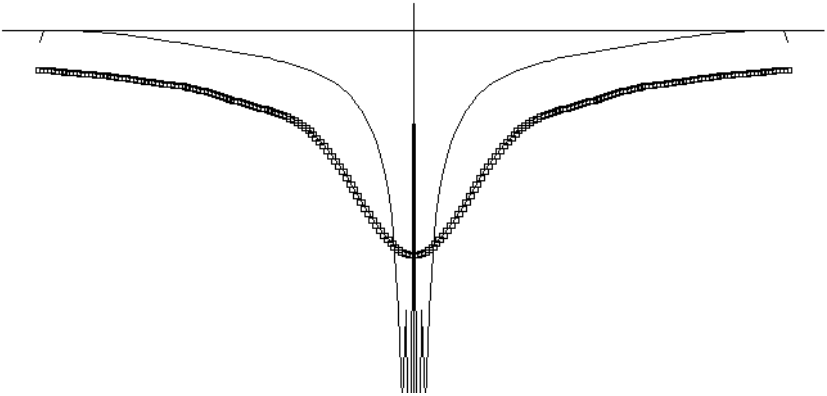

Figure 11

Tone Ether

If the harmonics of Figure 4 (right) are used to construct surface spherical harmonics on a sphere the distribution shown in Figure 12 is obtained.

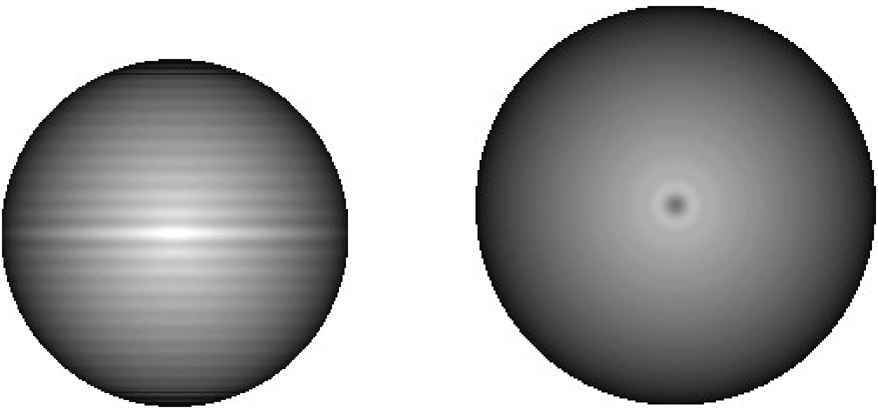

Figure 12

The right hand picture shows a polar view of the distribution, that on the left an elevation with the ‘equator’ along the middle. It is postulated that these may be seen as the ‘music’ of the tone ether arising in the body of the water when the vortex is destroyed by reversing the direction of stirring. Form harmonics are transformed into musical ones, which may indicate how the establishment of a vortex relates the fluid to the cosmos, which is then ‘caught’ in the ether.

Conclusions

A deep connection between egg and vortex path curve forms is revealed by means of Legendre transforms, the one being the transform of the other. Indeed an egg profile has Legendre coefficients with a harmonic profile that is a vortex. Idealised harmonic envelopes give a better fit than those calculated directly, although it should be noted that the use Simpson’s method of numerical integration for calculating the Legendre coefficients may account for some inaccuracy in the latter.

By varying the number of coefficients making up the egg form of the harmonic envelope, more realistic profiles are obtained. Practical work will be undertaken to see how well they fit real vortices, to supplement that already done for idealised path curve vortices.

The use of pivot-transform profiles is at an early stage of investigation and work is in hand to fit more diverse types of profile, as reducing λ below –5 leads to envelopes not covered by Figure 8 but which may well be more generalised such transforms. This involves optimising a number or parameters simultaneously, one of which enters the equations transcendentally. A suggestion made by Martin Levin was invaluable in solving this problem.

These harmonics may be used in the surface spherical harmonics postulated to arise in the tone ether when a stirred vortex is repeatedly reversed in direction as in the production of biodynamic preparations.

References

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