<u>Coordinates in Projective Geometry</u>

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A non-metric approach to projective geometry requires the assignment of coordinates to points on a line. Since cross-ratio (CR) is usually defined in terms of coordinates e.g. Ref 1, CR cannot be used to assign coordinates. If it is so used then CR must be defined in some other way.

Given a line, we may arbitrarily choose the zero, infinite and unit points O I and U with coordinates 0, ∞ and 1 respectively. If CR is somehow defined (e.g. metrically!) then of course the coordinates of some X may be assigned as its CR wrt OIU e.g. (OI, UX). If CR is defined in terms of coordinates then it may not be so used. So, given a visible point X on the line, how do we assign its coordinate? Or, given a coordinate other than -1, how do know which actual visible point it relates to? (a simple construction answers that for CR = -1).

In Ref 2 nets of rationality are discussed which do not require use of the Fixed Point Theorem (i.e. if three points of a line are self-corresponding for a transformation σ, then σ is the identity transformation). The transition to real coordinates requires either V&Y's Assumption P i.e. the Fixed Point Theorem becomes an axiom, or else the use of axioms of continuity and order, or else the use of CR. Even for a net of rationality the assignment of coordinates is unclear without the use of CR.

In Ref 3 a point X on a line AB is said to be xA+yB where A and B are reference points and x and y are numbers (type not stated). Since CR is there defined in terms of these coordinates, it cannot be used to assign coordinates. So it is unclear <u>where</u> X is visibly located on the line, or what value x/y takes for some given visible point.

If some function or other is used to define the coordinate of X given O I and U, how does that function relate to the position of a visible point on the line? My sneaking suspicion is that for all the rhetoric, Euclidean length or distance are implicitly assumed to define location.

Since CR may be used to prove the Fixed Point and Fundamental Theorems, it seems that the non-Pappian cases do not actually arise. Coxeter (Ref 4) points out that if the number basis is a division ring instead of a field then Pappus' Theorem is false. But that only applies <u>if</u> it is made clear how any number of any type is related to a location on the line. Also what kind of geometry arises, if any, when the number system is a division ring?

I have not found answers to these questions in the cited references.

<u>References</u>

1. Mihalek *Projective Geometry and Algebraic Structures*

2. Veblen and Young *Projective Geometry*

3. Faulkner *Projective Geometry*

4. Coxeter *Introduction to Geometry*