

Exploring Negative Space

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This paper explores an approach to science based on the interaction of two distinct spaces in contrast to the usual assumption of a single complicated manifold. The idea follows on the work of Steiner, Adams and Edwards (Refs 1 and 5). Cayley showed (Ref. 4) that a metric geometry may be derived from projective geometry by selecting an invariant conic section, which may be extended to an invariant quadric surface in three dimensions. The distance between two points A,B is then defined by relating it to the cross ratio (AB,IJ) where I,J are the points in which the line AB intersects the quadric Q. Cayley's formula for the distance AB is then

$$\frac{\log(\text{cross ratio})}{2i}$$

This particular expression is chosen so that Pythagoras' theorem is satisfied in the case of Euclidean geometry. The identical expression is used to calculate the angle between two planes α and β , where the cross ratio is that of α,β and the two tangent planes to Q in the line (α,β) .

In differential calculus the expression

$$ds^2 = g_{ij}dx_i dx_j \quad (1)$$

gives the infinitesimal distance ds between two neighbouring points where g_{ij} is the metric tensor. At a given point g_{ij} evaluates to a symmetrical matrix which represents Cayley's metric quadric at that point. The quadric thus varies across the manifold, but may not change its signature. The tensor g_{ij} enables non-parallel lengths and distances to be compared, which is not possible in affine geometry. The Euclidean metric tensor is given by $g_{ij} = 1$ when $i=j$ and 0 otherwise, which represents an imaginary quadric as the points lying on it are given by $x^2 + y^2 + z^2 + w^2 = 0$. Since the distance between two points one of which lies in Q is infinite we must assume Q lies at infinity and take $w=0$, so that Q is now the imaginary circle (sometimes referred to as a disc quadric)

$$x^2 + y^2 + z^2 = 0 = w \quad (2)$$

Adams (Ref. 1) was led to consider a space given by the polar quadric Q', which in plane co-ordinates has the same matrix, representing an imaginary cone with a real point O for its vertex. Instead of interpreting the metric (1) as relating to points he took it to be an equation in plane co-ordinates, which determines the separation of planes in the space it defines. It is clear that this does not represent an angle as in Euclidean geometry, for it becomes infinite for two planes one of which contains O. This is clearer if (1) is re-expressed as

$$d\tau^2 = g_{ij}dz_i dz_j \quad (3)$$

where τ is the unit of separation and z_i is a plane co-ordinate. Then for two finitely separated planes this is integrated to give τ , and is infinite if one of the two planes contains O. If we take the covariant form of (1) we obtain plane co-ordinates in Euclidean geometry, but we have not done this to emphasise that we are now considering a different kind of space for which the co-ordinates of planes are contravariant, and their separation plays an analogous role to that of distance in space. Under these assumptions we have to digest the idea of a new kind of metric in which planes are separated by a quantity that, unlike angle, may become infinite. We will refer to this new quantity

as *turn*, which is polar to distance in space. It is clear for similar reasons as for Euclidean space that O is at infinity, so that instead of having a plane at infinity we have a point at infinity. Whereas the plane at infinity in Euclidean space is experienced as being infinitely far away in an outer sense, O is experienced as being infinitely far away inwardly; an infinite inwardness.

Ref. 7 shows that for Euclidean geometry Cayley's formula for the distance between two points x_i, y_i reduces to

$$s^2 = \frac{(x_0 y_3 - y_0 x_3)^2 + (x_1 y_3 - y_1 x_3)^2 + (x_2 y_3 - y_2 x_3)^2}{x_3^2 y_3^2}$$

the dual of which for two planes u_i, v_i in counterspace is

$$\tau^2 = \frac{(u_0 v_3 - v_0 u_3)^2 + (u_1 v_3 - v_1 u_3)^2 + (u_2 v_3 - v_2 u_3)^2}{u_3^2 v_3^2}$$

If v_i is the plane at infinity then $v_3=1$ and $v_i=0$ otherwise, and for $u_i=(u_0,0,0,1)$ this reduces to

$$\tau = \pm u_0$$

Now in Euclidean geometry the plane co-ordinate $u_0 = -1/x_0$ where $(x_0,0,0,1)$ is the foot of the perpendicular from O to the plane, so that for generalised scaling we have

$$r \cdot \tau = \text{constant} \quad (4)$$

where r is the length of the radial normal to the plane and τ is its turn from Ω .

The separation of two points A,B in negative space is polar to that of planes in space, which we deduce behaves like an angle. We will call it *shift*. The Euclidean expression for the angle between two planes z_i, w_i (Ref 7) is

$$\cos(\theta) = \frac{z_0 w_0 + z_1 w_1 + z_2 w_2}{\sqrt{(z_0^2 + z_1^2 + z_2^2)(w_0^2 + w_1^2 + w_2^2)}}$$

The polar of this for two points x_i, y_i in negative space is:

$$\cos(\sigma) = \frac{x_0 y_0 + x_1 y_1 + x_2 y_2}{\sqrt{(x_0^2 + x_1^2 + x_2^2)(y_0^2 + y_1^2 + y_2^2)}}$$

where σ is the shift. Since this is the inner product of the two points regarded as end-points of two position vectors, we see that it is like the angle subtended by them at O. This is a useful intellectual "crutch", but it should be borne in mind that there are no angles in negative space. Thus two points on a line through O are separated by a shift of zero, and are "parallel". We will use the term polar-parallel for such points. We begin to see how different is negative space from Euclidean space. We could of course have applied (1) to this space to calculate the distance between points instead of following the route we did. There is no *a priori* mathematical reason to choose the one rather than the other, it being a matter of consciousness which we choose. We postulate that a scientific

approach to another kind of consciousness is conceivable on this basis, namely one for which a point may be experienced as the infinitude instead of a plane. Is such a space a mere mathematical curiosity, or could it have real significance? The thesis of this paper is that it could be. The motive lies in the above comments about consciousness, for we might be able to accommodate those experiences sometimes reported of other states of consciousness without the risk of becoming nebulous in our thinking.

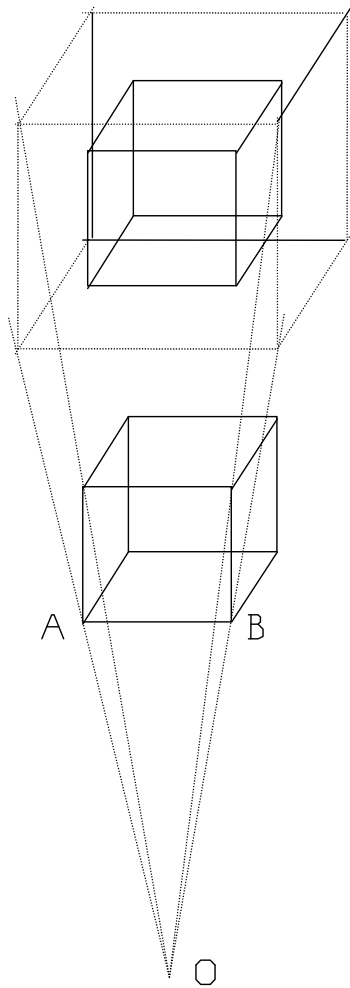


FIGURE 1

The central thesis of this paper is that objects may exist in both spaces at once, providing *linkages* between the two spaces. In the absence of such linkages **there is no relation whatever between the spaces**. We assume initially that points, lines or planes may exist in both spaces to form such linkages. Figure 1 shows the behaviour of a linked cube as it retreats from the counterspace infinitude O. The Euclidean behaviour needs no comment (solid cube), but the way it would behave in counterspace if it obeyed the metric of that space is shown by the dotted cube. This is most easily appreciated by considering the shifts involved, for in counterspace a metric transformation preserves shift and turn just as a Euclidean one in space leaves angle and length invariant. The shift σ between the points A and B equals the Euclidean angle they subtend at O, so that A and B must move outwards on their respective lines through O to preserve it. Thus the cube cannot obey the metrics of both spaces at once for the translation shown, and the second part of our thesis is that **the cube suffers strain as a result**. We mean this in the usual engineering sense when a body is deformed by an impressed force. This is a purely geometric quantity. When a body is subjected to strain e.g. a stretched steel wire, there is a resulting stress, and the third part of our thesis is that **the cube suffers stress as a result of the strain**. Then we make the transition from geometry to physics, just as Einstein had to introduce the energy/momentum

tensor. More generally we mean this for any linked object subject to transformation. Note that a rotation of the cube about an axis through O is strain-free, so we might expect such rotation to be encountered often in nature where stress-free transformations would be preferred.

Work will be done when the stress acts or is acted upon. Immediately we see that potential may be explained as a stress between space and counterspace. The stress may be suffered entirely in counterspace if the spatial metric is obeyed, or *vice versa*.

This rather simple approach to linkage will be refined as we go along, and first we need to consider how two entirely disjoint spaces may be related metrically. The roles of the infinite elements is

important as without an invariant infinity no measurements are possible e.g. in projective geometry. Thus for a linkage the infinite centre of counterspace must be related to a point of space, as implied in Figure 1, and the plane at infinity Ω of space must be linked. But, we cannot necessarily assume an extensive relation between the two spaces as there is no distance in counterspace, so the counterspace dimensionality is not coextensive with space, the only connection being the linked point. We thus assume that linkages of points are all in fact linkages to the counterspace infinite point, and this entails assuming that all such points are images of that infinitude. We refer to such points as counterspace infinitude images, or CSIs for short. If there are many linked points, then as they all image the same infinitude in counterspace we have what amounts to a fractal relation between the two spaces, as self-similarity is necessary from the perspective of every such point. This is particularly suitable for shift as it is evidently scale-invariant.

If a linked object does not have a proper fractal form (such as a Sierpinski sieve) then there will be fractal strain, which is more subtle than the simple kind of strain we considered initially, and as a result there will be fractal stress. The fruitfulness of what follows rests entirely on this notion.

If there are forces which are difficult to explain we might suspect that they result from stress in counterspace reacting back on objects in space via the linkage. The first such candidate considered at the outset of this work was gravity. This is treated in terms of linked points in which case we must analyse shift strain. As shift is not a vector (just as angle is not in space) we work with its gradient so that we have independence from the co-ordinates (throughout we will seek suitable tensors for linkages). It was found that gravity may be identified with the stress arising from the integral of the radial-rate-of-change of shift strain for two linked bodies, which gives the inverse square law and, with some further assumptions, Newton's Law.

Figure 2 shows a sphere with its CSI at the centre of gravity C, related to another CSI at O. We first consider a linked point P of the sphere, and note that there is shift strain caused by the fact that P is "seen" in different directions from O and C, the angle ϕ representing that strain. This is purely a result of the fractal assumption that both O and C are images of the infinitude of the same underlying counterspace, which thus suffers shift conflict in relation to P. We make the transition from geometry to physics by relating the resulting stress to the rate-of-change of the strain in the direction CO, choosing rate-of-change for the reason stated above. Taking $CO = r$, we differentiate ϕ wrt r , giving

$$\frac{d\phi}{dr} = \frac{\sin \theta}{d \cdot \cos \phi + r \cdot \cos \theta}$$

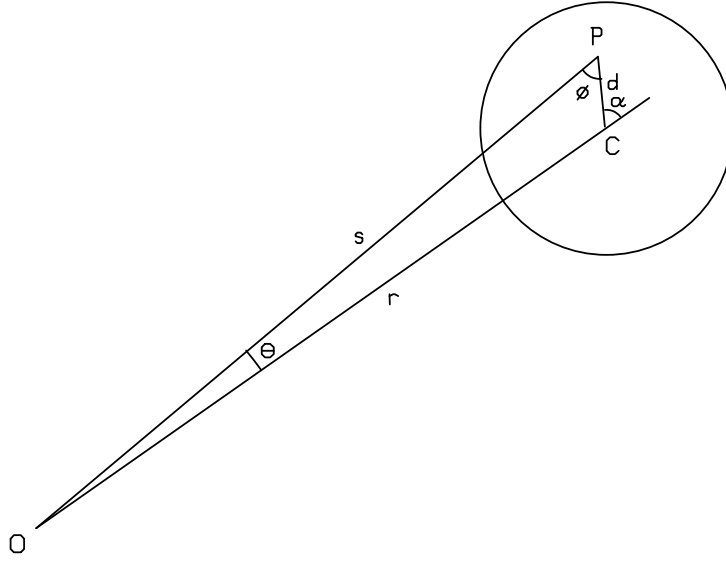


FIGURE 2

Using the identities

$$\sin \theta = \frac{d \cdot \sin \alpha}{s}$$

$$\cos \theta = \frac{r + d \cdot \cos \alpha}{s}$$

$$\cos \phi = \frac{d + r \cdot \cos \alpha}{s}$$

and the cosine rule for $s = OP$ we obtain

$$\frac{d\phi}{dr} = \frac{d \cdot \sin \alpha}{s^2}$$

This is the rate-of-change for the point P. If we consider all points such as P on the circle for given d and $\forall$ then we multiply by the circumference $2\pi \cdot d \cdot \sin \forall$, and our expression becomes

$$\frac{d\phi}{dr} = \frac{2\pi \cdot d^2 \cdot \sin^2 \alpha}{d^2 + r^2 + 2 \cdot d \cdot r \cdot \cos \alpha}$$

We integrate this wrt α i.e. we integrate the rate-of-change of shift strain $d\phi/dr$ wrt α , which gives the rate-of-change of shift strain for a spherical shell:

$$\frac{2.(r^2 - d^2).\tan^{-1}\left[\frac{(r-d).(\cos\alpha - 1)}{(r+d).\sin\alpha}\right] + (r^2 + d^2).\alpha - 2.r.d.\sin\alpha}{\frac{2.r^2}{\pi}}$$

Inserting the limits 0 to π , we obtain an indeterminate value for $\alpha=0$ which tends to zero, and for $\alpha = \pi$ we get

$$\frac{\pi^2 d^2}{r^2}$$

Finally we integrate this result for $r = 0$ to R , the radius of the sphere, to obtain the total rate-of-change of shift strain for the whole sphere, giving

$$\frac{\pi^2 R^3}{3r^2} = \frac{\pi V}{4r^2}$$

Thus we obtain the inverse square law, and a volume term. So far we have analysed the rate-of-change of shift strain in the direction of the line OC. To convert this to stress we must introduce a suitable scaling factor. What suggests itself is the density of the material making up the sphere, for if we include that at each point P in the integration then the volume V becomes the mass M:

$$\frac{\pi M}{4r^2} \tag{5}$$

The shift strain only exists in virtue of the CSI at O, about which we have made no explicit assumptions. In particular we have not assumed a mass there. Evidently the scaling must also be affected by this CSI, or the force would be independent of it, an aspect not yet included above. The simplest proposal is that the scaling of stress to strain gradient effected via the density depends upon the scaling between space and counterspace for the CSI at O i.e. a factor additional to the density enters in. However, we do not really know what we mean by ‘density’ in our present context, and it may turn out that it is itself a phenomenon associated with scaling. In fact this whole discussion suggests that.

If, however, there is also a massive body at O then (5) will apply to it too, and if the forces are to be equal and opposite (as we know they are from observation) then Newton’s Law follows, for if we denote by k_1 the effect on C of the scaling at O, and k_2 *vice versa*, then (5) is multiplied by k_1 and for equal forces we have

$$\frac{\pi k_1 M_1}{4.r^2} = \frac{\pi k_2 M_2}{4.r^2}$$

which is satisfied if $k_1 \propto M_2$ and $k_2 \propto M_1$. Thus we obtain Newton’s Law, the extra factors being absorbed by the gravitational constant G. Gravity thus has a definite explanation: it is caused by shift strain gradient.

This shows that ‘mass’ arises from the scaling between space and counterspace, which introduces an important aspect of the work to which we turn next. We have given enough detail for the reader to follow the argument, but which space will prevent for other results. This is to ‘ground’ the thesis and show how it works in one case.

Scaling is an important issue, for when a point or other element is linked to both spaces then a unit displacement in one space need not be accompanied by a corresponding unit displacement in the other. We have introduced one kind of scaling suitable for gravitation, but for other actions a more general one is required. We express it by the Jacobean matrix $\mathbf{J}$ which relates the rates of change of the three spatial co-ordinates to those of the counterspatial ones. This concerns infinitesimal changes. We also introduce at this stage the possibility that linkages need not be metric, but affine. We assumed a metric linkage for gravity, but extension of the work to gases, liquids, light and chemistry proved fruitful if affine linkages are also allowed. A general affine transformation permits expansion and contraction, leaves the plane at infinity invariant - and thus parallelism also - but does not preserve angle or distance. Ratios of lengths in a given direction are invariant, but line segments in distinct directions are not comparable. Ratios of volumes are also conserved. Special affine transformations have the additional property that volume is absolutely invariant. This suggests that special affine linkages are suitable for liquids which are approximately incompressible, while general affine transformations apply to gases which of course seek to expand. Thus we can relate the states of matter to these kinds of linkage:

metric	solids
special affine	liquids
general affine	gases.

If we assume a given scaling then we can derive the ideal gas law for gases by analysing *affine strain*, and many striking properties of liquids by analysing special affine strain between space and counterspace. This is described in detail in Ref. 6.

Returning to scaling, if we heat a solid it expands, which shows that a non-metric transformation is induced and the scaling between the two spaces is altered. This suggests that heat relates to the non-translatory aspects of scaling for which we use $\mathbf{J}$, and we have started with solids to emphasise the departure from the metric. Of course the same idea may now be applied to liquids and gases, and this proposal when applied to the analysis of gases leads from Boyle’s law to the ideal gas law. Thus it seems a fruitful idea. We make the simplest assumption that the terms of $\mathbf{J}$ are directly proportional to the temperature, which works for the ideal gas law and fortunately also implies that the viscosity of liquids should be temperature dependent. This scaling is locally variable, as it must be if heating one body is not to cause all others in the universe to expand in sympathy. The stochastic variation of the scaling we relate to *heat* as opposed to temperature, as that will introduce *scaling strain and stress*, the energy related to heat arising from the stress. This is explained in more detail in Ref. 6.

So far we have considered shift strain and stress, and the unifying assumption that the stress is directly proportional to the gradient of the strain proved fruitful for gravity, gases and liquids (in the case of gravity we implicitly resolved the gradient along the line joining the centres of gravity). If we consider strain arising from the linkages of planes we are led to *polar affine linkages* and *polar special affine linkages* related to counterspace. We can only very briefly sketch this aspect to indicate that it also is fruitful. The most suitable tensor for polar special affine linkages is a contravariant bivector in counterspace, which may be represented by a cone with a sense of rotation, a cone being polar to a circle. The bivector measures the *polar area* of the cone i.e. the

counterspace dual of spatial area. This proved fruitful for the treatment of light, where a photon is not thought of as a particle or a wave but as an affinely linked counterspace cone. Then we have constant polar area in one direction and a sense of rotation in counterspace, which parallel the conventional basic quantum state of a photon: energy, direction of propagation and polarisation.

For a *linked cone* we can consider the emission of a photon cone and we find that since turn is the reciprocal of radial distance, c.f. (4), the product $r \cdot \tau$ is constant, where r is the radial distance of the apex from the point of emission and τ the turn of the centre-plane of the cone (polar to the centre point of a circle) from the plane at infinity Ω . Just as areas may only be compared in affine geometry if they are coplanar, the dual in counterspace is that the *polar areas* of two cones may only be compared if their apices coincide. Thus an interaction of a photon cone must be at its apex, so no matter where the interaction occurs $t \cdot \tau$ has the same value. This suggested that *time be regarded as the reciprocal of radial turn* in which case we have $r/t = \text{constant}$, so if the scaling between space and counterspace is c we arrive at the constancy of the apparent velocity of light. However, since time is now measured by radial turn we cannot consistently suppose that the cone expands on emission (or two measures of time would be implied) but rather that there is an ensemble of possible coaxial cones, including the cylinder with the same polar area. An interaction singles out one such cone which has its apex at the point of interaction. Otherwise the cone becomes a photon cylinder which will be vanishingly thin and thus correspond to a ray. However, paradoxically it is not a ray of light *but a ray of darkness* as the polar area of the cylinder is “inside” it for counterspace i.e. outside it for space, so the light is all round the cylinder (its polar area is the integral of turn over all the planes that do not intersect its surface, dual to the integration of the area of a circle over its enclosed points). This also shows that light as such does not have a velocity, but the time at which an interaction occurs behaves as though it does, and the apparent “velocity” is constant. Thus many of the paradoxical aspects of relativity may be dissolved. Applications of these ideas satisfactorily explain reflection, refraction (giving Snell’s law), diffraction and scattering. Further, the “spooky” character of beam-splitter experiments is more comprehensible as the photon cylinder “embraces” the whole apparatus with its polar area before interaction, and thus may be expected to “know in advance” the state of the apparatus. It will be clear that the wave/particle paradox is not only dissolved by this approach, but does not even arise.

We can only mention here that the above approach to time finds corroboration when forces and potential are analysed, and the spherical wave equation is properly obtained from it.

For polar special affine geometry, constant polar volume spheres are considered, and actions within the surface of such a sphere are now time-invariant. In other words movements of tangential planes, or polar vectors in the surface, are time invariant. Such transformations are best analysed in terms of surface spherical harmonics if Laplace’s equation is to be satisfied, which brings us into direct contact with quantum physics. We arrive at the need for quantisation and the existence of non-physical waves, but now these are seen as belonging to counterspace, so it is not surprising that attempts to regard the wave function as physical failed at an early stage in the development of quantum physics. At first sight this seems best to apply to chemistry, and chemical bonds may be approached where two CSIs are at the foci of a spatial prolate spheroid which then acts as a sphere in counterspace (see Ref. 6) so that the same surface spherical harmonics are shared, giving a strain-free bond unless the CSIs are displaced from the foci. It appears at present that this is the covalent bond. Only 180° or 120° rotations are permitted in polar special affine geometry, which seem to relate to spins of $1/2$ or $1/3$, as in the case of the covalent bond described the linkages have an opposite sense so that “spin” is not a rotation but a permanently acting orientation of the linkage between the two spaces.

Three dimensional spinors are being studied as suitable tensors for metric linkages between space and counterspace as they may be interpreted as transformations of the absolutes (imaginary circle for space and imaginary cone for counterspace), having the same formal expression for both spaces. $x^2 + y^2 + z^2 = 0$ represents an isotropic vector which relates to a spinor, and in point co-ordinates this also represents the spatial absolute (c.f. (2) above), while in plane co-ordinates it represents the polar absolute of counterspace.

Spinors thus may be seen as again representing a permanently acting orientation of the linkage, but being metric they are not restricted regarding angle. Spinors are purely metric and have no affine counterparts (Ref. 3), so they are unsuitable for affine linkages. Thus when a measurement is made we propose that an affine situation is rendered metric (measurable) and spinors will then play a role. Since an affine linkage is more free than a metric one we expect that only one aspect of the linkage is measurable at a time due to the greater constraints imposed by the resulting metric linkage, which suggests an alternative approach to Heisenberg's uncertainty principle.

Bohm (Ref 2) and others sought for "hidden variables" or some other way of circumventing the Copenhagen interpretation of quantum physics, but without real success as in all cases only the physical/spatial aspects were considered. The above briefly sketched thesis shows that their aim may be realisable, not in terms of physical hidden variables but rather counterspace ones.

The theoretical investigation described in this paper is far from complete and requires further investigation by more qualified workers than the author. It has turned out to be a large work beyond the means of a single individual, but the aim of Ref. 6 is to show that it is feasible and potentially fruitful. It requires radical rethinking of many of our ideas, but without wishing to nullify the great progress made in this century. This short paper can only be descriptive, but some detail wrt gravity has been included as an example to illustrate how the ideas are applied. The strenuous attempt to understand matter in terms of particles has itself suffered ever greater 'strain' this century, and an attempt to construct a different approach from the ground up instead of relying on Bohr's correspondence principle seems worth while.

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