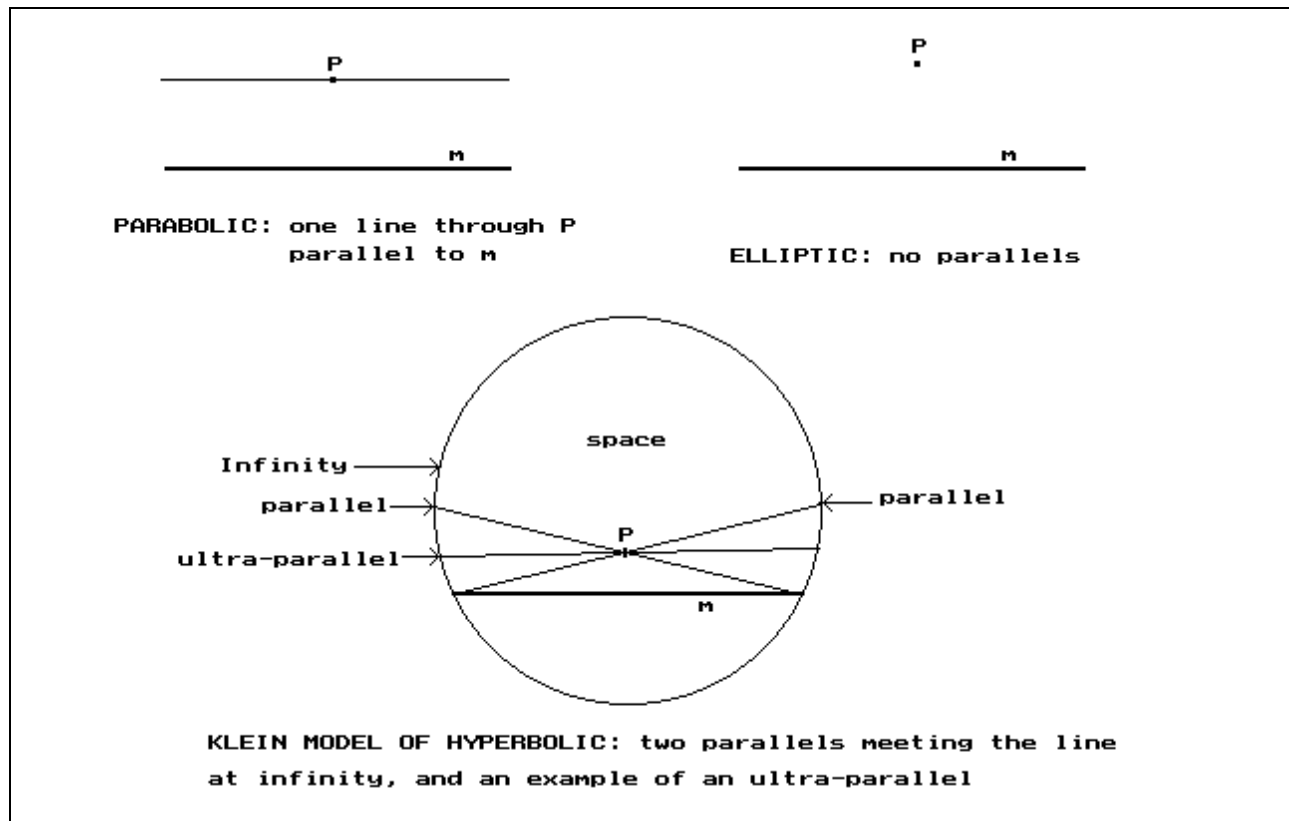


RETHINKING PHYSICS

Nick Thomas

What is it like inside the Sun? In Rudolf Steiner's time the current model involving the burning of hydrogen through nuclear fusion to produce helium had not been developed extensively. Steiner maintained that were it possible by some extraordinary means to travel to the inside of the Sun a big surprise would be in store, for far from finding burning gas we would - he said - find less than nothing, not just negative matter (which is now known to physics) but negative *space*. George Adams and Louis Locher-Ernst found a way to describe such a space based on geometry, and a possible application of that to physics will be sketched in this article.



Euclid's geometry is profoundly beautiful and satisfying, but it assumes implicitly that space is of a certain kind, and in particular that our intuitive notions of parallelism are valid in it. For ordinary common sense parallel lines do not meet, and neither do parallel planes. Furthermore, given a line and a separate point, then only one line exists passing through that point which is parallel to the first (Ptolemy). Ever since Euclid's time mathematicians were unhappy about this apparently obvious axiom, but until the last century none could find any way of proving or disproving it. Gauss was the first to solve the problem, but he dared not publish, so the honour went to Bolyai and Lobachevsky who independently solved it about 40 years after Gauss. The solution was far-

reaching: there can be more than one kind of geometry, and that studied by Euclid is a very special one in which the axiom is true, but there exist other geometries where it is false. Either there are no parallels at all through the point (elliptic geometry), or there are two with all lines in between being so-called ultra-parallels (hyperbolic geometry). Euclid's lies on the border between these two with just one parallel (parabolic geometry).

These ideas were not at first welcomed, but the matter was settled in 1868 when E. Beltrami proved that if these strange geometries are inconsistent then so is Euclid's. Felix Klein gave an alternative approach illustrated above. The scene was set for the development of non-euclidean geometries which have become well known through their use in Relativity Theory. The latter took the further step of assuming that a non-euclidean geometry is not only a mathematical construct, but may also describe the real world of physics i.e. space itself. A central idea has been that of a *manifold* in which the nature of space varies from place to place, sometimes being "flatter", sometimes more "curved". Euclid's geometry is a uniformly "flat" geometry by contrast. However, the assumption is still made that there is *one* space. Steiner and Adams suggested otherwise.

The study of projective geometry since the 15th Century prepared the way for this revolution, but projective geometry is *not* a non-euclidean geometry, but rather an *archetypal* one. Euclid's geometry is a metric geometry, and this distinction is most crucial in what follows. A metric geometry preserves certain types of measure as invariants, and generally non-euclidean geometries do so too. Projective geometry does not do this, for the only quantity it preserves is a so-called *cross-ratio*. To appreciate this it is helpful to notice an important difference in the way modern geometry is conceived which differs from Euclid's approach. The latter studied fixed forms such as triangles and circles, and investigated their relationships. The former studies *transformations*. Relativity Theory is founded on such an approach. We know that a mirror may distort the scene reflected, or transmit it faithfully. Thus an initial form may retain its proportions or have them altered. If the mirror magnifies then we have a transformation that leads to an expansion. The study of perspective led to the development of projective geometry, for we all know from first hand experience that the world does not *appear* to us as it actually *is*. When observing a cube rotate we all know that actually it keeps constant its volume, the length of its sides and the angles between its lines. Also the areas of its faces. But *that is not what we observe!* We need to distinguish here between our concept of euclidean space and what we actually see, for euclidean space is in fact a concept. Perspective allows us to represent three dimensional scenes in two dimensions, and to explain the laws which render what we know to be the case into the form which we actually observe (trees apparently getting smaller down an avenue etc.). The rotation of a cube is regarded, in modern geometry, as a transformation which can be precisely described mathematically. This kind of transformation, which leaves lengths and angles (and volumes and areas) unchanged, is referred to not surprisingly as a euclidean transformation, and further the kind of space in which such a transformation is possible is a euclidean space. A well known example of a non-euclidean transformation is that postulated by Einstein when he asserted that a body travelling close to the speed of light becomes shorter i.e. various lengths, angles, areas and the volume all change. This way of

thinking about geometry is absolutely fundamental to what will follow here.

Projective geometry studies transformations which change all measures e.g. a shadow of a triangular road sign thrown by a street lamp on the road has different proportions from the sign itself. Projective geometry does preserve straightness though i.e. if the road is flat then if an edge of the sign is straight, so will be its image in the shadow. One important property of projective geometry is that it moves infinity about! If you travel in a train you observe that nearby objects appear to go past quicker than more distant ones, while the Moon does not appear to move at all. You are undergoing a euclidean transformation while travelling in the train (which is not harmful to health, if you have not thought like that before), and this transformation leaves things infinitely far away unmoved, as suggested in an approximate way by the Moon. Were you to undergo a projective transformation it almost certainly would be harmful to health, and you might suffer the trauma of seeing infinitely distant stars rushing up close. Quite reasonably such behaviour is regarded as eccentric, so in order to move closer to our world mathematicians restrict the full range of transformations available in a projective world to those which leave infinity well alone! The resulting geometry is called *affine* geometry. Now parallel lines are said to meet at infinity, so if infinity stays put then lines that are parallel before a transformation remain parallel after it, even if they have changed in other ways. Like projective geometry, affine geometry (remember we are now regarding a geometry as characterised by certain kinds of transformations) changes lengths, angles, areas and volumes, but it has a healthy respect for infinity. While in euclidean geometry the cube remained the same size, an affine cube can expand or contract, but parallel faces remain parallel. Unlike the euclidean cube, however, lines in different directions need not remain in the same ratio, so the cube could become a prismatic shape after an affine transformation, with sides in different directions changing proportion. Also its volume could change. Now there exists a special version of affine geometry where volume becomes invariant (but not proportion). For example if a column of mercury in an inverted test tube on a table is released by withdrawing the test tube, it will certainly change shape, but its overall volume remains constant. This is called *special affine geometry*. It is most important for physics to note that lengths in different directions cannot be compared in any type of affine geometry, so scalar products have no meaning in this geometry. If a force is applied to a cart at an angle to its possible direction of movement, the work done in moving it is the scalar product of the force and the displacement. Such a calculation is only possible in metric geometry, but not in affine geometry.

Finally we may restrict the allowable transformations to those which leave right angles invariant, and also the ratios of lengths in different directions. We then end up with euclidean geometry. Technically this is accomplished by leaving an imaginary circle in the plane at infinity invariant. We find two major kinds of invariant measure: length and angle. Also volume and area.

When the possibility of non-euclidean metric geometries was realised, Cayley replaced the above steps by a single one: choose a quadric surface to be invariant (spheres, hyperboloids, ellipsoids and paraboloids are examples). The resulting restriction on projective geometry gives some kind of metric geometry. In the case of euclidean geometry the quadric is

degenerate, consisting of a special imaginary circle in the plane at infinity. A central thesis of this paper is that this "shortcut" hides important things, and that the four steps enumerated above are significant in themselves. They were

1. Start with projective geometry.
2. Obtain affine geometry by making a plane invariant (the plane at infinity).
3. Obtain special-affine geometry by disallowing expansion (or contraction).
4. Obtain metric geometry by choosing the so-called *absolute imaginary circle* in the plane at infinity.

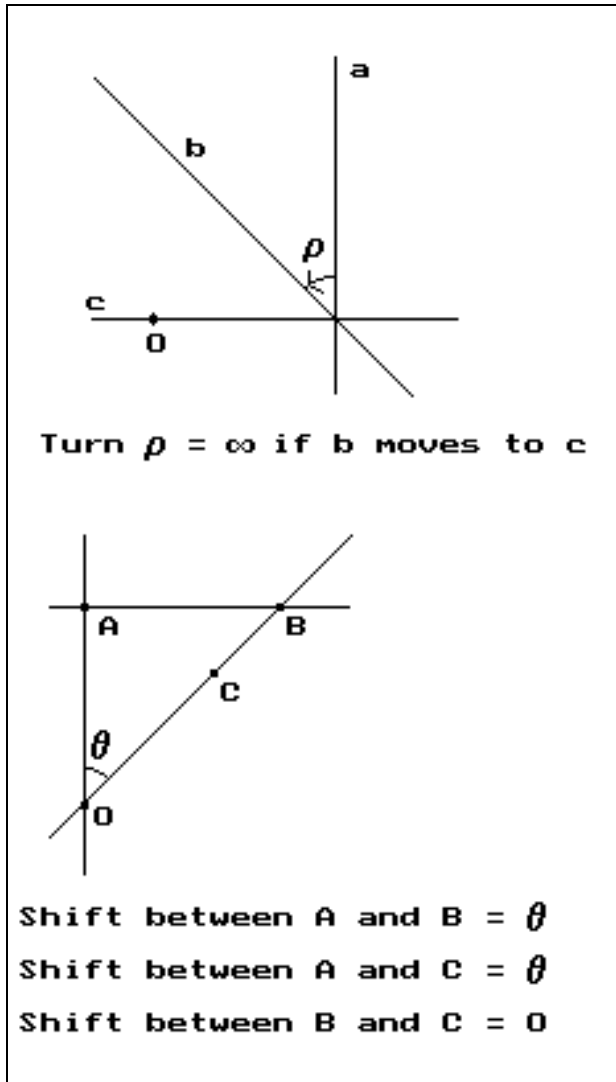
George Adams interpreted Steiner's negative space as the polar opposite or dual of Euclid's. Thus instead of an imaginary circle in an infinite plane he chose an *absolute imaginary cone in an infinite point*. Just as an imaginary circle (with equation $x^2+y^2+r^2=0$) nevertheless lies in a real plane, so an imaginary cone (with a similar equation in terms of planes) has a real vertex. Clearly it requires a different consciousness to be able to experience a point as being at infinity, to experience an *infinite inwardness* instead of an infinite outwardness as is normal. Such a space, it seems, Steiner had in mind for the interior of the Sun.

Here we will split this process up into four stages as we did for euclidean geometry:

1. Start with projective geometry.
2. Select a point at infinity as invariant, to give polar-affine geometry.
3. Restrict transformations to those where there is no expansion or contraction, to give special polar-affine geometry.
4. Select the special imaginary cone with its vertex in the infinite point as invariant, to give polar-euclidean geometry which characterises counter-space. It is a metric geometry.

What, in the resulting metric geometry, correspond to length and angle? We must be quite clear that counter-space is a *metric* space in contrast to polar affine space, so what is measured in it? We can only state the result here, which is derived from Cayley's formula for deriving length and angle from cross ratio for a general metric geometry. Applied to euclidean geometry we get the well known formula $d^2 = x^2+y^2+z^2$ for length, and a formula for the cosine of the angle between two planes as a scalar product of the plane coordinates. We are now faced with an interesting decision, for we may follow the relativity-style route and seek a metric which defines a distance between any two points of counter-space. Or, we dualise this which gives a measure for the separation of any two planes which is dual to distance. There is no purely mathematical reason for choosing the one route rather than the other; it is a matter of consciousness. Here we are exploring the second route, for which the metric for planes is quite

unlike an angle, varying from 0 to 4, the latter when one plane lies in the infinite point. Also it is a vector, unlike the angle between two planes in euclidean geometry. We will refer to this measure as *turn*, so two planes in counter-space are separated by a definite turn rather than an angle. Similarly the dualisation leads to the separation of two points in counter-space as being like an angle since it varies cyclically from 0 to 2π , which we will refer to as *shift*. It is not a vector.



Two points on a line through the infinite point have a zero shift (reminiscent of null lines in Minkovsky's metric), which is dual to the fact that two planes in euclidean space sharing a line in the plane at infinity are parallel and so the angle between them is zero even though they are distinct. The shift between two points is easy to visualise if we imagine two lines joining each point to the counter-space infinite point, for then the shift equals the angle between those lines. This at first sight strange measure is critically important for an understanding of gravity, as we shall see.

The four steps from projective geometry to euclidean geometry (which we will simply refer to as "space" henceforth) are suggestive of the four states of matter, for the metric step (4) clearly describes solids, while special affine geometry relates to liquids which are incompressible (ideally), having as it does the property of constant volume but plasticity otherwise. General affine geometry allows expansion and so may relate to gases. Heat cannot simply be related to projective geometry, however. It is further interesting that three dimensional special affine geometry does not induce special affine geometry in planes, but general affine geometry, which can be seen to

relate to the gas like behaviour at the surfaces of liquids (evaporation).

The three steps to counter-space may thus relate to the ethers, with polar affine geometry related to light, special polar affine geometry to chemical action, and counter-space itself to life processes. This leads to interesting insights when followed up. Before continuing with this we will now consider the central thesis of this paper.

If indeed counter-space describes an aspect of the real world then we may

ask what happens if an object is linked to both space and counter-space simultaneously? Consider a cube linked to both: if it undergoes a transformation then that transformation should be characteristic of both spaces at once i.e. lengths, angles, turns, shifts and volume should be invariant. If this is not so then we will get strain in one or both spaces, leading to stress i.e. force. If the cube behaves in a euclidean way then its lengths and angles will be invariant, but we then find that corresponding shifts and turns must change, so it becomes stressed in counter-space. Evidence of such a linkage might be sought in an otherwise inexplicable force, one notorious candidate being gravity. For this it is natural to examine shift in view of the point-centred quality of gravity, in which case we analyse the possibility that two points may retain their euclidean distance invariant but that the counter-space shift between them is forced to vary, giving rise to strain and hence a force. In fact gravity can be derived in this way, giving the inverse square law and proportionality to volume (and hence, via density, mass). It is too technical to describe in detail here, but further aspects will arise later.

This result gives strong support to the validity of the line of thought being developed, and in particular that counter-space relates to minerals as well as living beings, especially where shift is involved. A general principle to be followed up, then, is that scientific evidence for the etheric may be sought not in contradictions to physical law, but rather in an understanding of aspects of the world that were thought to be physical but in fact are rooted in this idea that two qualitatively different spaces are simultaneously active.

It is natural to ask if any transformations are stress-free in both spaces, for such are likely to be the equilibrium conditions sought by Nature. Only four have been found:

1. A rotation about an axis through the counter-space infinite point. This may explain the ubiquitous appearance of vortices and rotary motion in general. If there is a counter-space infinitude in the centre of the Earth, this would explain why water vortices tend to behave the way they do, with an axis tending towards a line through the centre, but with instability near the tip as tangent planes related to counter-space strive to avoid "going to infinity" (very large stress) which they would if they passed through the Earth's centre.
2. A polar transformation (where points and planes are related via a quadric surface) has the property that a transformation in one space yields a stress free polar transformation in the other.
3. A reflection.
4. It is possible also to regard uniform rectilinear motion as nearly stress free given certain assumptions about the relative scaling of spatial and counter-spatial measures, but acceleration is not stress-free.

If a transformation in one space causes stress in the other, we see that work must be done to bring about the transformation. The result depends upon the situation, but broadly we can envisage two possible outcomes: first that we are left with stress in one (or even both) spaces, which is

potential energy; or secondly that the relation to the spaces is changed to become stress free, in which case absorption or emission of energy must account for that change.

So far we have only considered a fully metric linkage between space and counter-space, relevant to solids. The above description of the elements may now be taken further by taking into account the possibility that a linkage between space and counter-space may be affine, or polar affine, as well as fully metric. Further, it is important to note the distinction Steiner pointed to in the *Light Course* between the kinematic and the dynamic, for force and mass cannot necessarily be accounted for by geometry alone. In the present context we may postulate that mass is the content of a linkage between space and counter-space where the spatial metric dominates, and ether is the content where the counter-space metric dominates. The quality of these contents is not exhausted by the geometry which provides the stage, as it were, upon which matter and ether as actors play their part. In particular we should not confuse ether with counter-space. Goethean observation is necessary to win the content for which the geometry can provide a non-materialistic context. Both are needed to win through to a Michaelic science which does full justice both to the qualitative and the quantitative.

A question naturally arises at this point: how many counter-spaces are there? It is possible to consider a counter-space as fractally related to space i.e. that images of the counter-space infinitude appear in space due to a fractal linkage of the two spaces. Thus if a point is related to two such counter-space infinitude images (CSIs) a stress may arise if it then appears to have two different positions for the primal counter-space.

Returning to the Sun, spectroscopic analysis reveals hydrogen and helium and other trace elements on its surface. This does not prove that any such elements exist inside the Sun. Steiner spoke of a "tearing" of space e.g. in lightning (*Heat Course*), and we may consider the Sun's corona also as a result of such a "tearing", although we are not suggesting the corona is lightning. If there is a large stress between space and counter-space on the Sun's surface the result may be a "tearing" which results in a fractal relation between the counter-space of the Sun and ordinary space. The meaning of the "tearing" is just this fractal multiplication of CSIs. Gas is the result of this fractal "tearing", and also heat and light. The result is an affine linkage between space and counter-space expressed in fractal form. Dually there arises a polar affine linkage related to light (but which may not be fractal). Both linkages are expansive. Now an affine linkage cannot give rise to metric stress since metric quantities are not invariants, but as suggested above a point can be inconsistently related to the primal counter-space, giving stress. We will refer to this as *affine stress*. Regarding a CSI as such a point the only way this stress can be relieved is for the CSI to move away from the others. Hence we have gas pressure if it is in a metric container where the affine and metric behaviours must harmonise. Furthermore, since different directions are incomparable in affine space we can perhaps see why the kinetic theory of gases works, which considers pressure to arise from the rate of change of momentum of individual atoms travelling independently each in a definite direction (we are not espousing this view here). An analysis of the gradient of the affine strain for a point-triangle gives a force which is inversely proportional to the linear size of the triangle and passes

through its circumcentre. We take triangles rather than solid structures because the affine quality restricts us to one counterspatial "direction" at a time. Summing the effects of all such triangles of CSIs in a volume of gas we find the total force on the outer surface of the metric container to be inversely proportional to its surface area, and hence in total we get pressure inversely proportional to volume. Finally, if the scaling between space and counterspace to obtain stress from strain is proportional to temperature we obtain the ideal gas law $PV=RT$. The important issue of how the two spaces are scaled is illustrated here, and fundamental physical constants seem to relate to this. In particular we see how heat enters into the picture in relation to scaling, noting also its effect on expansion and contraction of bodies, which are non-euclidean transformations. If we have a given gas such as hydrogen we suppose that it has a primal counter-space of its own which becomes fractally related to space when ponderable and in the gaseous state. This agrees with Steiner's description of the existence of elements throughout the cosmos even when not materially expressed. To summarise, gas is composed of multiple CSIs under affine stress causing pressure.

When we come to the liquid state we find that we need an understanding of an aspect of gravity to explain it, so we will first consider the solid state. Only in solids do we have metric stress, and in particular we find particles suffer shift stress in counter-space, which is a development of affine stress in so far as the fractal coupling gives shift stress. A detailed calculation shows this to manifest as a force like gravity. The interesting thing is that this only appears for solids, as shift stress is not possible in affine or polar affine space, agreeing with Steiner's indication in the *Heat Course* that gravity does not affect a pure liquid. Crystal structure arises in solids through a principle of least action, where hexagonal, cubic etc. forms share and hence economise on action, a process which makes and breaks counterspace linkages. Apart from these static forms, one other class of structure seems to satisfy this requirement, namely path curves. This may explain why path curves appear in plants, and are related to geometric transformations, and yet plants do not appear actually to grow in that way.

We must account for liquid in such a way that it does have weight, as is observed. Water tends to form short range structures, and on this basis we can solve the problem for water, for if short range crystal forms occur then gravity is ushered in for those micro-structures. We envisage the fluid state as an equilibrium between the crystalline and the gaseous, between gravity which is contractive and the expansive affine stress of gas. Then special affine geometry with its constant volume character is an expression of this equilibrium. This approach indicates that a liquid must be extensively microstructured in view of its weight. However, this is not the only way of resolving the problem, and a fuller consideration of what we mean by "mineral" is required. We may perform an analysis of affine strain for a constant volume tetrahedron (following the logic applied to gases but now in special affine space), which results in a gradient which is only zero for a regular tetrahedron, and is inversely proportional to its linear size. The forces on the vertices act parallel to the opposite faces, giving shear effects. This manifests mainly in the surface of the liquid as the effects balance out in the centre of a liquid mass. We also find that a small base combined with a distant fourth vertex results in the base moving towards the vertex, not vice versa. This accounts for the

tendency of a liquid drop to form a sphere. Surface tension can be seen to arise from a combination of behaviours of tetrahedra containing CSI vertices in the surface. The rich range of action of various forms of such tetrahedra seems to account well for the properties of a liquid. An interesting thing that emerges is that an analysis of affine strain is identical to that of shift strain were it to exist in liquid, and the same applies to gases. We are thus led to propose that the analysis applies to the point-wise mineral qualities of the liquid, to which gravity may also apply. The non-mineral aspect related to the constant volume property upon which the analysis is superimposed, to which gravity does not apply, is perhaps what Steiner had in mind in the *Heat Course*.

Looking at the dual situation for chemical action where we have a special polar affine linkage, we envisage an equilibrium between "short range" structure in counter-space and light (dual to gas). But what does "short range" mean in counter-space? Put quite simply it means far from infinity, so we are concerned with structures "far" from the CSI, whereas "long range" means "closer". To clarify, if we have a plane linked to both spaces then if the line through the CSI perpendicular to it (in the euclidean sense) is of length r then it can be shown that the turn of that plane in from the euclidean infinite plane is $\tau = s/r$ where s is a scaling factor between the two spaces. "Far" from the CSI means τ is small, whereas "close" means τ is large and r is small. This may mirror the distinction between the nucleus of an atom where counter-space entities and structures are "long range" and the electron shells where they are relatively "short range". s needs to be determined, but this indicates it is a very small number. The difference from conventional atomic theory is *that we regard the structure and properties of elements to arise from counter-space structures rather than from particles*, in line with Steiner's indication that where we think of atoms we should look for cosmic activity.

Again, as we found for gases, we start to see (dimly as yet) why atomic theory arises and why it works as a model. A cardinal question concerns the nature of the counter-space structure of a primal element. Remarkable woven patterns of planes with respect to their mutual turns suggest a structure of light, again reminiscent of Steiner's characterisation of matter as "woven light". Briefly, it is possible to construct lattices of parallel planes such that the turn between two parallel planes equals that between a plane and one rotated through a definite angle in the next array.

The structure is dynamic and contains a weaving between spiral planar movements and radial parallel arrays. Hexagonal and triangular prisms are interesting as they sit in balance with infinity, having the same turn between neighbouring planes as each plane has to the plane at infinity, showing a possible transition from a mobile affine structure to a static metric one. However this is only very tentative as yet. What is clear is that "long range" counter-space structure concerns the identity of a primal element while "short range" structure concerns chemical activity, and dual to liquid there is an equilibrium between the chemical-element-forming tendency and the expansive tendency of light characteristic of polar affine space.

We have not yet considered light. It has posed some of the greatest riddles to physics this century, so we need not expect an easy answer. Light arose with gas in early evolution which is why we look to a polar affine linkage between space and counter-space dual to the affine linkage

characterising gas. There is no pointwise linkage here, so the light source CSI is not located in space, which expresses the conventional idea that position may be indeterminate. Thus a plane in this counter-space is also indeterminately located, and for space only has an orientation (expressed as a line in the euclidean plane at infinity). A more detailed account requires a closer study of polar-area i.e. the dual in counter-space to area in space. While light itself is essentially etheric, when it relates to darkness in the form of photons we obtain not particles but cones in counter-space. We get neither particles nor waves but a polar area which can express the experimental properties of photons. The fact that a polar area appears extended over a volume in euclidean space may help explain some of the apparently paradoxical multi-path experiments carried out with photons where the latter appear to be in several places at once. In fact they are, but not as particles or waves. A strong initial support for this approach is that it unexpectedly yielded Balmer's formula for the emission of photon cones. Reflection, refraction and diffraction can be understood, and Taylor's version of Young's double-slit experiment (more recently repeated with electrons and very recently with atoms) can be explained without paradox.

Chemical action seems to involve bonds mediated by photon cones, and as planar linkages are very "stiff" these cones do not detach easily, which is what constitutes a "bond". A stronger bond may be associated with an interesting geometrical property. Quadric surfaces cannot change "signature" in affine or metric space as their relation to infinity is invariant i.e. a surface either intersects the plane at infinity or it does not (or it touches). This relation is referred to conventionally as the signature. Thus an ellipsoid does not intersect the plane at infinity, but a hyperboloid does, so the one cannot be transformed into the other by means of a real transformation. For counter-space a similar signature arises in relation to the CSI (0 say), for a quadric surface may have a real tangent cone in 0 in which case it is a "hyperboloid" for counter-space even if it looks like an ellipsoid to a euclidean consciousness. If it has no real tangent cone in 0 then it is an "ellipsoid" for counter-space, so what looks like a euclidean hyperboloid may have a finite polar-volume, which has been confirmed by integrating polar-area over that polar-volume. The point is, this can explain why matter does not collapse, for constituent euclidean spheres would have to change signature in counter-space for this to happen. Conversely very strong bonds are possible if signature change is required to separate the constituents of matter. It is not yet clear whether this distinguishes covalent from metallic and hydrogen bonds, or whether it goes beyond chemistry to nuclear forces.

A subtle interpretation of time is needed fully to appreciate what is said above, which explains the constancy of the velocity of light, in the sense that a photon cone apex appears to have that velocity but not in fact the light itself. Quantisation arises as a necessity, to avoid infinite time intervals, together with what otherwise requires relativity i.e. these two aspects are quite naturally united. It also explains the sense in which we have a time organism when we consider life ether, and why the apparently "rigid" quality of counter-space as a metric space, apparently so un-life-like, does in fact express life. The implications of this approach to time are currently being explored, so these brief comments must suffice.

The "fallen ethers" have yet to be satisfactorily included, although a

promising start has been made with magnetic and electric polarisation, which points the way. Briefly, the idea is that a scaling quadric forming part of a CSI may be ellipsoidal at the metric level, and thus polarised. The fractal interaction of the metrically linked members of a population may "entrain" the shapes of its affine members (for which polarisation is otherwise indeterminate), which is referred to as "affine entrainment". An example is a capacitor for which the plates are metrically polarised, the dielectric being affinely entrained into polarisation. It seems at this stage that "fields" are accounted for by the propagation of fractal effects in this way, which seems more satisfactory than fictitious "lines of force".

The indeterminacies in quantum physics seem to arise when a transition from affine to metric space occurs, for a non-strained affine configuration may end up strained if forced by a measurement to become metric. Thus at the affine level we have indeterminacy from a metric standpoint, and only one of the complementary quantities (e.g. position and momentum) can be realised metrically with full precision.

This article is necessarily aphoristic and much will seem to be mere assertion, which is unavoidable in a short account like this of what has become a large work. It is planned to remedy this in the form of a book where the accompanying mathematics and diagrams can be presented. We are concerned with ongoing research which is developing week by week, and what has been described may have changed considerably by then. The hope was to convey something in a short article of the whole picture that is developing.

References

"Heat Course", Rudolf Steiner, lectures given 1 to 14 March 1920

"Light Course", Rudolf Steiner,
lectures given 23 December 1919 to 3 January 1920
Goethean Science Foundation

"Space and the Light of Creation", George Adams Kaufmann
Published by the Author

"The Plant Between Sun and Earth", Adams and Whicher
Rudolf Steiner Press

"Raum und Gegenraum", Louis Locher-Ernst
Philosophisch-Anthroposophischer Verlag am Goetheanum

"Projective Geometry" by Lawrence Edwards
Floris Press

"The Vortex of Life", Lawrence Edwards
Floris Press

"Analytic and Projective Geometry" by Dirk J. Struik
Addison-Wesley Publishing Co. Inc.

"Projective Geometry" by T. Ewan Faulkner
Oliver and Boyd